The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2002–2003

APPLIED ALGEBRA FOR ENGINEERS

Time allowed TWO hours

Candidates must NOT start writing their answers until told to do so.

This paper has TWO sections which carry equal marks.

Section A ***comprises TWELVE multiple-choice questions.***
Responses must be made on the response sheet provided.

Section B ***comprises FOUR questions.***
Full marks may be obtained for THREE complete answers.
Credit will be given for the best THREE answers.

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a dictionary to translate between that language and English provided that neither language is the subject of this examination. No electronic devices capable of storing and retrieving text may be used.

DO NOT turn examination paper over until instructed to do so

You **MUST NOT** remove the question paper.
Failure to comply may result in the award of a mark of zero.
On this cover sheet, enter your **NAME** and your **SCHOOL**.

CANDIDATE'S NAME (in block capitals) .....

SCHOOL .....

Instructions for answering the multiple-choice questions

(a) Responses will be read by a machine. You MUST NOT mark the response sheet in any way other than as indicated on the response sheet.

(b) All rough work should be within the examination book; rough work will not be used for assessment.

(c) You MUST record exactly one response for each question; choose E if you wish to abstain. (Each response is marked +3 if correct, −1 if incorrect, and 0 for abstain. The total is scaled.)

(d) On the response sheet:

- Please use an HB pencil.
- Mark your answer with a single horizontal line.
- If you make a mistake, erase it completely.
- Do not mark with ticks, crosses or circles.
- Do not forget to write your NAME and MODULE details.
- Do not forget to enter and code your CANDIDATE NUMBER.
- Your candidate number is your Student Identity Number with the addition of 00 at the end. For example if your Student ID is 9076421 your candidate number is 907642100
- Mark the box corresponding to your School in the section headed 'Other Information' as follows:

School	Code	School	Code
SChEME	A	Civil Engineering	D
Electrical and Electronic Engineering	B	Mechanical, Materials, Manufacturing, Engineering and Management	E
Built Environment	C	Other Courses	F

SECTION B

13 For each of the following systems of equations

(1) express the system in matrix form;

(2) construct the augmented matrix;

(3) use the method of ROW REDUCTION to obtain the set of solutions, where it exists.

(a)
$$\begin{aligned} x + 2y - z &= 2 \\ x + y + z &= 3 \\ 3x + 2y + z &= 1 \end{aligned}$$

(b)
$$\begin{aligned} x + y + 4z &= 3 \\ 2x + y + 2z &= 4 \\ 3x + y &= 5. \end{aligned}$$

14 Show that the eigenvalues of the matrix

$$\boldsymbol{A} = \begin{pmatrix} 2 & -3 & 1 \\ -3 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix}$$

are $\lambda = 0, 1, 9$.

Find the eigenvectors corresponding to $\lambda = 0$ and $\lambda = 1$.

15 A radio transmitter T is situated at the point with coordinates $(2, 2, 0)$, in appropriate units, with respect to cartesian coordinates $Oxyz$ where z is vertically upwards. Its signals are detected by receivers A and B each situated on the top of a building.

If A has coordinates $(4, 3, 1)$ and B has coordinates $(-3, 1, 2)$ find :

(a) the position vector of the point Q on TA such that $TQ = \frac{2}{5}TA$;

(b) the equation of the plane, labelled π, containing A, B and T;

(c) the perpendicular distance of O from π.

16 (a) Two rockets R_1 and R_2 move in space in such a way that at time t their position vectors are $\boldsymbol{r}_1(t)$ and $\boldsymbol{r}_2(t)$ respectively, given by

$$\boldsymbol{r}_1(t) = t^2\boldsymbol{i} + 2t\boldsymbol{j} + 5\boldsymbol{k} \quad \text{and} \quad \boldsymbol{r}_2(t) = (4t - 3)\boldsymbol{i} + 2t^2\boldsymbol{j} + 5\boldsymbol{k}.$$

Determine the time $t = t'$ at which the rockets collide (i.e. reach the same point in space at the same time).

Show that at $t = t'$:

(i) the velocities of the rockets are in the same direction but that R_2 has twice the speed of R_1;

(ii) the accelerations of the rockets are at right angles and that the magnitude of the acceleration of R_2 is twice that of R_1.

(b) Show that the two planes

$$\begin{aligned} 2x &+ 8y + 3z = 0 \\ x &+ 2y + z = 0 \end{aligned}$$

intersect in the line L_1, which has vector equation

$$\boldsymbol{r} = \lambda(1,\ \tfrac{1}{2},\ -2) \quad \text{for} \quad -\infty < \lambda < \infty.$$

Find the coordinates of the point of intersection of L_1 with the line L_2 which has vector equation

$$\boldsymbol{r} = (1,\ 2,\ 3) + \mu(1,\ 1,\ -\tfrac{1}{3}) \quad \text{for} \quad -\infty < \mu < \infty.$$