The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING 2003–2004

APPLIED ALGEBRA FOR ENGINEERS

Time allowed TWO hours

Candidates must NOT start writing their answers until told to do so.

This paper has TWO sections which carry equal marks.

Section A ***comprises TWELVE multiple-choice questions. Responses must be made on the response sheet provided.***

Section B ***comprises FOUR questions. Full marks may be obtained for THREE complete answers. Credit will be given for the best THREE answers.***

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a dictionary to translate between that language and English provided that neither language is the subject of this examination.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so.

You **MUST NOT** remove the question paper.
Failure to comply may result in the award of a mark of zero.
On this cover sheet, enter your **NAME** and your **SCHOOL**.

CANDIDATE'S NAME (in block capitals) .....

SCHOOL .....

Instructions for answering the multiple-choice questions

(a) Responses will be read by a machine. You MUST NOT mark the response sheet in any way other than as indicated on the response sheet.

(b) All rough work should be within the examination book; rough work will not be used for assessment.

(c) You MUST record exactly one response for each question; choose E if you wish to abstain.
(Each response is marked +3 if correct, −1 if incorrect, and 0 for abstain. The total is scaled.)

(d) On the response sheet:

- Please use an HB pencil.
- Mark your answer with a single horizontal line.
- If you make a mistake, erase it completely.
- Do not mark with ticks, crosses or circles.
- Do not forget to write your NAME and MODULE details.
- Do not forget to enter and code your 7 digit STUDENT ID.
- Mark the box corresponding to your School in the section headed 'Other Information' as follows:

School	Code	School	Code
SChEME	A	Civil Engineering	D
Electrical and Electronic Engineering	B	Mechanical, Materials, Manufacturing, Engineering and Management	E
Built Environment	C	Foundation Year	F
Other Courses	G		

Section B

13 A submarine situated at the origin of cartesian coordinates $Oxyz$, where z is vertically upwards, observes three stationary enemy submarines A, B and C with position vectors $(1,\ 2,\ 5)$, $(2,\ 1,\ 7)$ and $(-3,\ 3,\ 6)$ respectively in appropriate units.

Find

(a) the vector equation of the line through A and B.

(b) the cartesian equation of the plane P containing A, B and C.

(c) the angle between P and a horizontal plane.

14 Express the system of homogeneous equations

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ 2x_1 + x_2 + (\rho+1)x_3 &= 0 \\ \rho x_1 + 3x_2 + 2x_3 &= 0 \end{aligned}$$

in the matrix form $\boldsymbol{AX} = \boldsymbol{0}$.

Determine the two values of ρ for which the system has non-trivial solutions.

Find all the solutions corresponding to each value of ρ.

15 Show that the eigenvalues of the matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 1 \end{pmatrix}$$

are $\lambda = 1,\ -1,\ 5$.

Find the eigenvectors corresponding to $\lambda = 1$ and $\lambda = -1$.

16 Three planes have vector equations

$$\boldsymbol{r}\cdot(-1,\ 3,\ -1) = 7, \quad \boldsymbol{r}\cdot(0,\ -1,\ 2) = -3 \quad \text{and} \quad \boldsymbol{r}\cdot(2,\ -1,\ 0) = 1.$$

Write these equations in cartesian form and use the ROW REDUCTION method to find the position vector of the point Q where the three planes intersect.

The points A, B, and C have position vectors $(7,\ 5,\ 1)$, $(3,\ 5,\ 1)$ and $(3,\ 5,\ 5)$ respectively. Find the volume of the tetrahedron with vertices at Q, A, B and C.