The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING 2004–2005

APPLIED ALGEBRA FOR ENGINEERS

Time allowed TWO hours

Candidates must NOT start writing their answers until told to do so.

This paper has TWO sections which carry equal marks.

Section A ***comprises TWELVE multiple-choice questions. Responses must be made on the response sheet provided.***

Section B ***comprises FOUR questions. Full marks may be obtained for THREE complete answers. Credit will be given for the best THREE answers.***

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so.

You **MUST NOT** remove the question paper.
Failure to comply may result in the award of a mark of zero.
On this cover sheet, enter your **NAME** and your **SCHOOL**.

CANDIDATE'S NAME (in block capitals) ....

SCHOOL ....

Section A

Section A Multiple Choice Questions are not released - a specimen Section A is available on Melees.

Section B

1 (a) Test the following system of homogeneous equations to determine whether it has non trivial solutions,

$$\begin{array}{rcrcrcl} 2x_1 & + & 5x_2 & & & = & 0 \\ -x_1 & + & 3x_2 & - & 2x_3 & = & 0 \\ 3x_1 & + & 13x_2 & - & 2x_3 & = & 0. \end{array}$$

Determine all the solutions , if any exist.

(b) Express the following system of equations in matrix form

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ 2x_1 & + & 5x_2 & + & 3x_3 & = & 0 \\ x_1 & & & + & 8x_3 & = & 5. \end{array}$$

Construct the augmented matrix for the system and use the ROW REDUCTION method to obtain the solution.

[Note: no marks will be given for a solution by any other method.]

2 A coastguard helicopter A observes a sailing boat as it approaches the finishing line of a race. The finishing line is the line joining two points B and C situated on either side of a harbour. If the position vectors of A, B and C are $(-2, 1, 3)$, $(20, 5, 0)$ and $(25, 10, 0)$ respectively (in appropriate units) with respect to cartesian coordinates $Oxyz$, where z is vertically upwards, find:

(a) the Cartesian equation of the plane containing A, B and C.

(b) the vector equation of the line passing through B and C.

The boat is observed to have position vector $(10, 3, 0)$ and to be travelling parallel to the vector $(3, 1, 0)$. If it is assumed that the boat does not change direction find:

(c) the vector equation of the straight line path of the boat

(d) the position vector of the point where the boat crosses the finishing line.

3 Show that the eigenvalues of the matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

are $\lambda = 1, -2, 3$.

Find the eigenvectors corresponding to $\lambda = 1$ and $\lambda = -2$.

4 (a) Determine whether the following system of non-homogeneous equations has either a unique solution, no solution or an infinite number of solutions.

$$\begin{aligned} 2x_1 - 3x_2 + 3x_3 &= 3 \\ 4x_1 + 6x_2 + 9x_3 &= -1 \\ 2x_1 + x_2 + 4x_3 &= 1. \end{aligned}$$

[You do NOT need to find the solutions, if they exist.]

(b) A particle moves in space such that at time t its position vector $\boldsymbol{r}(t)$ is given by

$$\boldsymbol{r} = \left(e^t + e^{-t}\right)\boldsymbol{i} + \left(e^t - e^{-t}\right)\boldsymbol{j} + t^2\boldsymbol{k} \qquad \text{for} \quad t \geq 0.$$

Determine the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ of the particle and show that

(i) $\boldsymbol{r}$ and $\boldsymbol{v}$ are orthogonal vectors at $t = 0$.

(ii) $\boldsymbol{v}$ and $\boldsymbol{a}$ are orthogonal vectors at $t = 0$.

(iii) $\boldsymbol{v}$ and $\boldsymbol{r} - \boldsymbol{a}$ are orthogonal vectors at $t = 0$ and $t = \sqrt{2}$.