The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING 2005–2006

APPLIED ALGEBRA FOR ENGINEERS

Time allowed TWO hours

Candidates must NOT start writing their answers until told to do so.

This paper has TWO sections which carry equal marks.

Section A ***comprises TWELVE multiple-choice questions.***
Responses must be made on the response sheet provided.

Section B ***comprises FOUR questions.***
Full marks may be obtained for THREE complete answers.
Credit will be given for the best THREE answers.

An indication is given of the approximate weighting of each section of a question by means of a figure enclosed by square brackets, eg [12]*, immediately following that section.*

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so.

You **MUST NOT** remove the question paper. Failure to comply may result in the award of a mark of zero. On this cover sheet, enter your STUDENT ID and your SCHOOL.

STUDENT ID . . . .

SCHOOL . . . .

Instructions for answering the multiple-choice questions

(a) Responses will be read by a machine. You MUST NOT mark the response sheet in any way other than as indicated on the response sheet.

(b) All rough work should be within the examination book; rough work will not be used for assessment.

(c) You MUST record exactly one response for each question; choose E if you wish to abstain. (Each response is marked $+3$ if correct, -1 if incorrect, and 0 for abstain. The total is scaled.)

(d) On the response sheet:

- Please use an HB pencil.
- Mark your answer with a single horizontal line.
- If you make a mistake, erase it completely.
- Do not mark with ticks, crosses or circles.
- Do not forget to write your NAME and MODULE details.
- Do not forget to enter and code your 7 digit STUDENT ID.
- Mark the box corresponding to your School in the section headed 'Other Information' as follows:

School	Code	School	Code
SChEME	A	Civil Engineering	D
Electrical and Electronic Engineering	B	Mechanical, Materials, Manufacturing, Engineering and Management	E
Built Environment	C	Other Courses	F

Section A

1 The value of the determinant

$$\begin{vmatrix} 2 & -1 & 3 \\ -4 & 2 & 3 \\ -2 & 5 & 0 \end{vmatrix}$$

is

(a) -96, (b) -84, (c) -72, (d) 108.

2 The force F which has magnitude 6 and direction parallel to the vector $(3,1,-1)$ has components

(a) $\left(\frac{18}{\sqrt{11}}, \frac{6}{\sqrt{11}}, -\frac{6}{\sqrt{11}}\right)$, (b) $\left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)$,

(c) $(3\sqrt{11}, \sqrt{11}, -\sqrt{11})$, (d) $(18, 6, -6)$.

3 The angle between the vectors $\boldsymbol{u} = 2\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ is

(a) $\cos^{-1}\left(\frac{-20}{7\sqrt{17}}\right)$, (b) $\cos^{-1}\left(\frac{-8}{7\sqrt{17}}\right)$, (c) $\cos^{-1}\left(\frac{-20}{\sqrt{119}}\right)$, (d) $\cos^{-1}\left(\frac{-8}{119}\right)$.

4 If the position vectors of the points A and B are $(4,-1,3)$ and $(10,8,-2)$ respectively, then the position vector of the point P on the line joining A to B such that $AP = \frac{1}{3}AB$ is

(a) $\left(12, 11, -\frac{11}{3}\right)$, (b) $\left(2, -4, \frac{14}{3}\right)$, (c) $\left(8, 5, -\frac{1}{3}\right)$, (d) $\left(6, 2, \frac{4}{3}\right)$.

5 If $\boldsymbol{u} = (2,3,1)$ and $\boldsymbol{v} = (-1,4,-3)$, then $\boldsymbol{u} \times \boldsymbol{v}$ has components

(a) $(-5,5,11)$, (b) $(13,-5,-11)$, (c) $(-13,5,11)$, (d) $(-13,-5,-11)$.

6 For any two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, the expression $(6\boldsymbol{p} + 3\boldsymbol{q}) \times (\boldsymbol{q} + 2\boldsymbol{p})$ can be simplified to

(a) $6\boldsymbol{p} \times \boldsymbol{q}$, (b) $12\boldsymbol{p} \times \boldsymbol{q}$, (c) $-12\boldsymbol{p} \times \boldsymbol{q}$, (d) $\mathbf{0}$.

7 The cartesian equation of the plane which is parallel to the vectors $\boldsymbol{i} + \boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j}$ and passes through the point $(3,1,1)$ is

(a) $2x + y + z = 8$, (b) $x - y - z = 1$, (c) $3x + y + z = 1$, (d) $x + y + z = 5$.

8 If the matrices A, B and C are defined by

$$A = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ -1 & -1 \end{pmatrix} \quad \text{and} \quad C = AB$$

then the element C_{22} is

(a) 0, (b) 4, (c) 6, (d) 7.

9 If the matrices A, B and C are defined by

$$A = \begin{pmatrix} 2 & x \\ 2x & 1 \end{pmatrix}, \quad B = \begin{pmatrix} y & 2 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad C = A + B$$

where x and y are constants, then C is an anti-symmetric matrix only when

(a) $x = 1$ and for all values of y, (b) $x = 3,\ y = -2$,

(c) $x = -1,\ y = -2$, (d) $x = -1$ and for all values of y.

10 If the matrix A and its inverse A^{-1} are defined to be

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad A^{-1} = \begin{pmatrix} -\frac{1}{2} & x & \frac{1}{2} \\ 1 & 0 & 0 \\ y & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

where x and y are constants, then

(a) $x = -\frac{1}{2}, y = -\frac{1}{2}$, (b) $x = -\frac{1}{2}, y = \frac{1}{2}$, (c) $x = \frac{1}{2}, y = -\frac{1}{2}$, (d) $x = \frac{1}{2}, y = \frac{1}{2}$.

11 Given that $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ are eigenvectors of the matrix $\begin{pmatrix} 2 & -3 \\ -10 & 1 \end{pmatrix}$, the corresponding eigenvalues are

(a) $\frac{1}{4}$ and $\frac{1}{7}$, (b) $-\frac{1}{4}$ and $\frac{1}{7}$, (c) 4 and 7, (d) -4 and 7.

12 If the Jacobi iterative method is applied to the system of equations

$$\begin{aligned} 2x + y + z &= 3 \\ x + 2y + z &= 4 \\ x + y + 2z &= 1, \end{aligned}$$

starting from the initial guess $x = y = z = 1$, the result after one step of the iteration is

(a) $x = \frac{3}{2},\ y = 2,\ z = \frac{1}{2}$, (b) $x = 1,\ y = 2,\ z = -1$,

(c) $x = \frac{1}{2},\ y = 1,\ z = -\frac{1}{2}$, (d) $x = 3,\ y = 4,\ z = 1$.

Section B

13 A building has a triangular roof. The top of the roof is at the point A with coordinates $(5, 7, 10)$ and the other two corners of the roof B and C have coordinates $(0, 4, 6)$ and $(10, 4, 6)$ respectively.

(a) Find the area of the roof. [5]

(b) Find a unit normal vector to the roof. [3]

(c) Find the cartesian equation for the plane of the roof. [4]

A telephone wire runs in a straight line from the point $P = (12, 11, 14)$ to the top of the roof.

(d) Find the length of the wire. [3]

(e) Find the angle between the wire and the roof. [5]

14 (a) A bee flies in search of food so that at time t its position vector $\boldsymbol{r}(t)$ is given by

$$\boldsymbol{r}(t) = (2t + \cos t)\boldsymbol{i} + (t + \sin t)\boldsymbol{j} + (2 - \sin t)\boldsymbol{k}.$$

(i) Find the velocity $\boldsymbol{v}$ and the acceleration $\boldsymbol{a}$ of the bee. [4]

(ii) Find the velocity and speed of the bee at $t = 0$. [3]

(iii) Show that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{r}$ at $t = 0$. [3]

(iv) At what time does the bee experience the maximum force? [3]

(b) If $f(x, y, z) = x^2y^2 - y^3 + xz$, and $\boldsymbol{u}$ is the gradient of f, $\boldsymbol{u} = \boldsymbol{\nabla} f$, find $\boldsymbol{u}$ and $\boldsymbol{\nabla} \cdot \boldsymbol{u}$ at the point $(1, 1, 2)$. [7]

15 (a) Show that the system of homogeneous equations

$$\begin{array}{rcrcrcl} x & - & y & - & z & = & 0 \\ 3x & + & y & & & = & 0 \\ 6x & - & 2y & - & 3z & = & 0 \end{array}$$

has infinitely many solutions, and determine the general solution in terms of a parameter t. [7]

(b) Write the system of equations

$$\begin{array}{rcrcrcr} x & + & 5y & - & 3z & = & 17 \\ 2x & + & 9y & - & 14z & = & 23 \\ x & - & 5y & + & 5z & = & -5 \end{array}$$

in matrix form.

Construct the augmented matrix and use the row reduction method to obtain the solution. [13]

[*Note: no marks will be given for a solution by any other method.*]

16 Show that the eigenvalues of the matrix

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 4 \\ 0 & 4 & 2 \end{pmatrix}$$

are $\lambda = 2, 7, -3$. [8]

Find the eigenvectors corresponding to $\lambda = 2$ and $\lambda = 7$ and show that they are orthogonal. [12]