HG1M02—Applied Algebra for Engineers

Coursework 3

To be handed in no later than Friday 27th April. Coursework **must** be handed in to one of the designated mathematics coursework collection points; **not** to me, and **not** to [any] school office. For more details, see overleaf.

1. For the scalar field f and the vector field $\boldsymbol{v}$ defined by

$$f(x,y,z) = x^3y^3z^3 \quad \text{and} \quad \boldsymbol{v}(x,y,z) = x^3y^2z\boldsymbol{i} + y^3z^2x\boldsymbol{j} + z^3x^2y\boldsymbol{k},$$

find *(a)* ∇f; *(b)* $\nabla\cdot\nabla f$; *(c)* $\nabla\cdot\boldsymbol{v}$; *(d)* $\nabla\times\boldsymbol{v}$.

2. Given the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & -1 \\ 4 & 1 & 4 \end{pmatrix},$$

find A^2 and hence the matrix $M = A^2 - 4A - 10I$, where I is the 3×3 identity matrix. [CHECK: the first column of M has the elements -12, -3 and 6, but not necessarily in that order.] Verify that $MA = -3I$, and hence write down the inverse matrix, A^{-1}, of A.

3. Use row operations to solve the system of equations

$$\begin{aligned} x + y + 2z &= 2, \\ 2x - y - z &= 1, \\ 4x + y + 4z &= 3 \end{aligned}$$

by the Gauss–Jordan process.

4. Explain how you could have used the matrix M found in Q2 to solve the equations of Q3, and show that this leads to the same answer for x, y and z.

Marks: Q1: 3 marks for each part, total 12. **Q2:** 3 marks each for finding A^2, M, MA and A^{-1}, total 12. **Q3:** 5 marks for the tableau, 3 for the solution, total 8. **Q4:** 4 marks for the explanation, 4 for showing that it works, total 8. Grand total: **40**.

Handing in coursework:

All mathematics coursework **must** be handed in to one of the **coursework collection points** in either the Pope building [near room B2] or the Mathematics and Physics building [in the far corner of the area at the top of the main stairs].

You need to use a **green** cover sheet [provided at the collection point]. Fill in sections A and C of the cover sheet, **time-stamp** the receipt [instructions at the collection point], tear off your recipt, and **staple** the rest of the sheet to your work [stapler provided]. These instructions are also given at the collection points. Make sure your name is on every sheet, in case of accident, and drop your work into the box. In section A, the bits you *need* to get *right* are your **name**, and the module **code** [HG1M02].

You have to sign that your work is not plagiarised. This means that it is your work, and is not copied from work done by someone else. There is no reason why you should not talk about the problems with your friends, but there is a clear moral difference between that and simply copying down their solutions. Plagiarists are not fit to be members of this or any other academic community, and plagiarism is a very serious offence.

You are reminded that this coursework counts $6\frac{2}{3}\%$ towards your assessment for this module. If you are late [without acceptable reason], you will lose 5% per working day; after five days, you will receive a mark of zero. Applications for any extension to the deadline should be made to me, and *must* be accompanied by a completed 'Extenuating Circumstances Form' giving your reasons for the application and signed by your personal tutor. You can get these forms from your school office or from your tutor. I will tell you whether or not your application will be allowed, and if so what your new deadline is; if you believe my decision to be unfair, then you may appeal to the Director of Mathematics Service Teaching, Dr S. Hibberd. — ANW