Section A

1 The value of the determinant

$$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

is

(a) -11, (b) -9, (c) -7, (d) 13.

2 The vector $\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k}$ has length

(a) 6, (b) $\sqrt{6}$, (c) 14, (d) $\sqrt{14}$.

3 The component of the vector $(-1,1,2)$ in the direction of the vector $(2,2,-1)$ is

(a) -2, (b) $-2/3$, (c) -6, (d) 6.

4 If the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, then it follows that

(a) $\boldsymbol{a}\cdot\boldsymbol{b} = 0$, (b) $\boldsymbol{a}\times\boldsymbol{b} = \mathbf{0}$, (c) either $\boldsymbol{a} = \mathbf{0}$ or $\boldsymbol{b} = \mathbf{0}$,

(d) the parallelogram based on $\boldsymbol{a}$ and $\boldsymbol{b}$ is a rhombus.

5 If $\boldsymbol{u} = (1,1,2)$ and $\boldsymbol{v} = (2,5,1)$, then $\boldsymbol{u}\times\boldsymbol{v}$ is

(a) $(-9,-3,3)$, (b) $(9,-3,-3)$, (c) 9, (d) $(-9,3,3)$.

6 If $\boldsymbol{a} = \boldsymbol{i}$, $\boldsymbol{b} = \boldsymbol{i}+\boldsymbol{j}$ and $\boldsymbol{c} = \boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$, then the scalar triple product $[\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}]$ is

(a) -3, (b) -1, (c) 1, (d) 3.

7 The plane through the point $\boldsymbol{p}$ and perpendicular to the vector $\boldsymbol{n}$ has equation

(a) $(\boldsymbol{r}-\boldsymbol{n})\cdot\boldsymbol{p} = 0$, (b) $(\boldsymbol{r}-\boldsymbol{p})\cdot\boldsymbol{n} = 0$, (c) $(\boldsymbol{r}-\boldsymbol{n})\times\boldsymbol{p} = \mathbf{0}$, (d) $(\boldsymbol{r}-\boldsymbol{p})\times\boldsymbol{n} = \mathbf{0}$.

8 If A is a 2×3 matrix, and B is a 2×4 matrix, then the following matrix products exist:

(a) AB, (b) BA, (c) $A^T B$, (d) none of these.

Turn Over

9 If the matrix A is defined by

$$A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

then it is *not* true that

(a) A is symmetric, (b) A has no inverse, (c) $A - A^T$ is a zero matrix,

(d) A is orthogonal.

10 If the matrices R, S and T are defined by

$$R = \begin{pmatrix} 1 & 4 & 2 \\ -1 & 0 & 3 \\ 5 & 2 & 1 \end{pmatrix}, \quad S = \begin{pmatrix} 5 & 7 & 1 \\ 0 & 3 & 2 \\ -6 & 2 & 4 \end{pmatrix}, \quad T = (t_{ij}) = RS,$$

then the element t_{32} is

(a) 11, (b) 4, (c) 43, (d) -16.

11 The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$ is

(a) $\begin{pmatrix} -1 & \frac{2}{3} \\ 1 & -\frac{1}{3} \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 1 \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$ (d) $\begin{pmatrix} 3 & -3 \\ -2 & 1 \end{pmatrix}$.

12 The eigenvalues of the matrix $\begin{pmatrix} 2 & 2 \\ 6 & 3 \end{pmatrix}$ are

(a) -6 and 1, (b) 6 and -1, (c) 2 and 3, (d) -6 and 6.

Section B

13 A cable is connected to a pylon at the point $A = (0, 0, 5)$ and runs in a straight line in the direction of the vector $(5,4,1)$. Another cable starts at the point $B = (0, 3, 0)$ and runs in a straight line to be attached to a vertical pole at $(4, -1, h)$, for some h.

(a) Find the equation of each cable in parametric form, [4]
and show that the cables intersect if $h = 12.8$. [4]

(b) In fact h is taken to be 8. Find a vector $\boldsymbol{v}$ which is normal to both cables for this value of h, [5]
and find the component of $\overrightarrow{AB}$ in the direction of $\boldsymbol{v}$. [4]

(c) For safety reasons, it is required that the second cable is at least a distance 1 from the first cable at the nearest approach. Find whether or not the chosen value of h is safe. [3]

14 (a) A particle of constant mass m moves so that its position vector $\boldsymbol{r}(t)$ at time t is

$$\boldsymbol{r}(t) = \cos(t)\,\boldsymbol{i} + \sin(t)\,\boldsymbol{j} + 2t\,\boldsymbol{k}.$$

Find the velocity $\boldsymbol{v}(t)$ of the particle, [3]
and show that it moves with constant speed $\sqrt{5}$. [2]
Find also the acceleration $\boldsymbol{a}(t)$ of the particle, [2]
and show that the force acting on the particle has magnitude m [1]
and acts through the point $(0, 0, 2t)$. [2]

(b) The temperature at the point (x, y, z) is given by

$$T(x, y, z) = x^2 y \sin z$$

over some region of space. Find the temperature gradient $\boldsymbol{u} = \nabla T$. [4]
Find also the Laplacian, $\nabla \cdot \boldsymbol{u} = \nabla^2 T$, [3]
and verify that $\nabla \times \boldsymbol{u} = \nabla \times \nabla T = \boldsymbol{0}$. [3]

15 (a) For what value of the parameter t do the homogeneous equations

$$2x + y + tz = 0,$$
$$x + 4y + z = 0 \text{ and}$$
$$3x - y - 10z = 0$$

have non-trivial solutions? [4]
Find a non-trivial solution for this value of t. [3]

(b) Write the system of equations

$$x - 4y - 2z = 21,$$
$$2x + y + 2z = 3 \text{ and}$$
$$3x + 2y - z = -2$$

in matrix form. [2]
Construct the augmented matrix for this system, [2]
and use the row reduction method [*and no other*] to obtain the solution. [9]

16 Find the three eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}.$$

[8]
Show that $(1, 3, 1)^T$ is an eigenvector of A corresponding to an eigenvalue 2, [4]
and find eigenvectors corresponding to the other two eigenvalues of A. [8]

End