Matrices

A matrix is simply an array or table of numbers. See the sections *A Summary of Matrix Algebra* and *A Summary of Special Matrices* in the course booklet.

For example,

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & -1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

are, respectively, 2×3 and 3×1 matrices, or matrices of order 2×3 and 3×1. For examples, we will be using small matrices, say 3×3; real problems may also use quite small matrices, *e.g.* to describe changes of co-ordinates in 3-dimensions in a systematic way; but often use large matrices, say 100×100 or even huge ones, say 10000×10000.

An example of a real-life application of very large matrices would be stock control: there could be a row for each product sold in a supermarket, and a column for each store, and the entries in the table could represent the number in stock, or the price, or the number sold.

The point of using a matrix is to bundle up its *elements* into one larger, compound object; so we will usually give it a name, such as M or C above. Commonly, the name is in upper case, and the corresponding lower-case letter, with subscripts corresponding to the row and column, is used for the elements. For example,

$$m_{11} = 1,\ m_{12} = 0,\ m_{13} = 2,\ m_{21} = 2,\ m_{22} = 1 \text{ and } m_{23} = -1.$$

Applications of matrices

- Simultaneous equations.
  — continuing the formalisation—compare determinants.

- Transformation of co-ordinates.
  — such as rotations in three dimensions. Computational geometry, CGI, robotics, ….

- Tables, spreadsheets, and similar.
  — but see the comment below.

- Connectivity and networks.
  — for example, $m_{ij} = 1$ if place i is connected by a direct road to place j, $m_{ij} = 0$ otherwise. Or m_{ij} could be the distance, or time taken, or electrical resistance, *etc.*, after which we can ask questions about global properties of the network.

- Tensors, such as stress/strain tensors.
  — describe a more general relationship between vectors, in which m_{ij} describes the effect on the i-th component of $\boldsymbol{u}$ of the j-th component of $\boldsymbol{v}$.

Beyond the present scope: There is no need for the elements of a matrix to be numbers; they could be any objects [people, functions, vectors, …]. But in practice we want to do mathematics with them, and this usually involves some concept of 'adding' or 'multiplying', *etc.*, so it is much more 'interesting' [mathematically] if the elements are things we already know how to do mathematics with.

Matrix addition

Two matrices can be added or subtracted if [and only if!] they have the same order [shape]. For example, given

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}; \; B = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & -1 & 2 \\ 5 & 0 & 3 \end{pmatrix},$$

then

$$A+B = \begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix} \text{ and } A-B = \begin{pmatrix} 0 & -1 \\ 3 & 2 \end{pmatrix}$$

[add or subtract corresponding elements], but [for example] $A+C$ does not exist.

Scalar multiplication

Any matrix can be multiplied by a scalar. For example,

$$3A = \begin{pmatrix} 3 & 0 \\ 6 & 12 \end{pmatrix}$$

[multiply each element by the scalar].

Transposition

Every matrix has a transpose, in which rows and columns are swapped. This is usually written with a superscript T; M^T is the transpose of M. Note that $m^T{}_{ij} = m_{ji}$, and that the order of M is equally swapped around. For example,

$$A^T = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix} \text{ and } C^T = \begin{pmatrix} 1 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}.$$