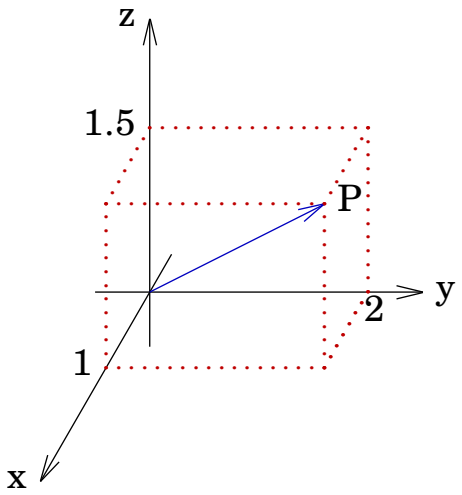
HG1M02—Applied Algebra for Engineers

Vectors

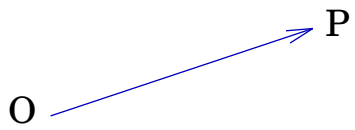
Vectors are *abstractions* of:

- lists of co-ordinates.



"P is the point with position vector $(1, 2, 1.5)$."

- Physical things with magnitude [size] and direction:



"P has position vector $\overrightarrow{OP}$ [or, in printing, $\boldsymbol{p}$ or, in handwriting, $\underline{p}$] relative to the origin O."

- These concepts are *different*, very *similar*, and very easily *confused*.

Co-ordinates

- Less ‘interesting’.
- More ‘practical’.
- Assume cartesian [(x, y, z)], so that co-ordinates represent components along three ‘orthogonal’ axes.
- Sometimes written as *row* vectors, sometimes as *column*:

$$(x, y, z) \text{ or } \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

  Differences unimportant at this stage.

- No reason other than [perhaps!] physical reality to have *three* components.
- Concept generalises to matrices, tensors, ….

Magnitude and direction *

- Rooted in physical reality
- Can, but mostly shouldn't, find components in order to convert to list representation.
- Independent of co-ordinate system, so we don't need to say what that is [until later …]. Theory applies to cartesians, polars, ….
- Laws of physics [and so of science/engineering in general] don't know what co-ordinates we have chosen; so vector laws can't depend on that. 'Galileian Relativity'.

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* Not the full story!

Examples:

Scalars

- Mass, density, time, temperature, speed, pressure, [electric] charge, ….

Vectors

- Displacement [position], velocity, acceleration, force, momentum, temperature gradient, electric field, ….

???

- Angle, angular velocity, area, colour, ….

  Area? It has a size [eg "1.23 square metres"] and a direction [that of the normal]. It has some of the properties of 'real' vectors, but not others. It is a 'pseudo-vector'. Likewise angular velocity [eg size 123 rpm, direction that of the axis of rotation]. On the other hand, angles and colours may have components, but they don't have other properties of vectors at all, really.

OK, we have all these vectors, …

…, so what can we do with them?

Algebra

- Add/subtract them.
- Multiply/divide them by scalars.
- Multiply them by other vectors, in two different ways.
- **But NOT divide them by other vectors.** Not ever, please.
- Solve vector equations.

Calculus

- Differentiate with respect to a scalar [eg time].
- Differentiate with respect to space, in various ways.
- Beyond the scope of this module: integrate, in various ways; solve vector differential equations.

See also …

… the ‘definitions’ in the module booklet.