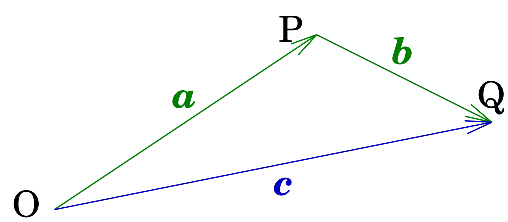
Vector Algebra

Addition

- *Components*: Add corresponding components:

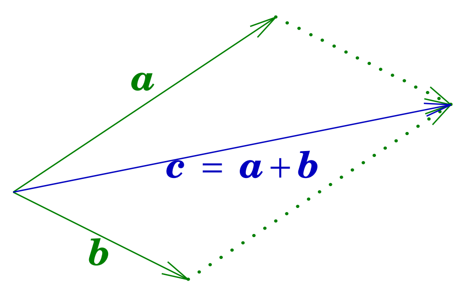
$$(1,2,2) + (3,-1,1) = (4,1,3).$$

- *Size / direction*: Triangle law:



$\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ or, in position vector form, $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$.

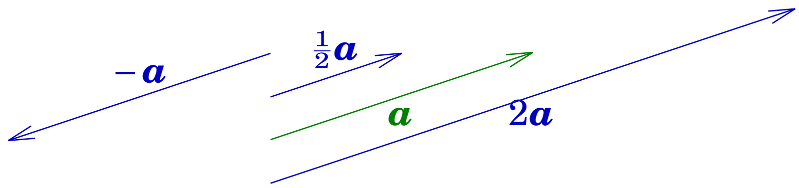
- or, equivalently, parallelogram law:



- *Subtraction*: Subtract corresponding components; or use $\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$, where $-\boldsymbol{b}$ is the vector with the same size as $\boldsymbol{b}$ but the opposite direction.

Scalar multiplication

- Multiply components: if $\boldsymbol{a} = (3, -1, 1)$, then $2\boldsymbol{a} = (6, -2, 2)$, $\frac{1}{2}\boldsymbol{a} = (1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$, and $-\boldsymbol{a} = (-3, 1, -1)$. Or scale size but keep direction, or reverse direction [for negative scalars]:

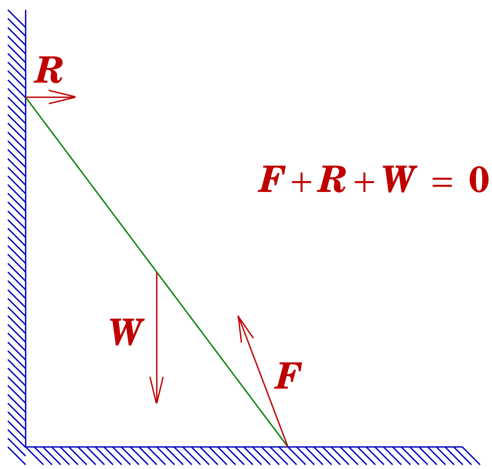


Examples

- *Triangle [polygon] of forces*
  If a body is in equilibrium, and is acted on by forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, ..., $\boldsymbol{F}_n$, then

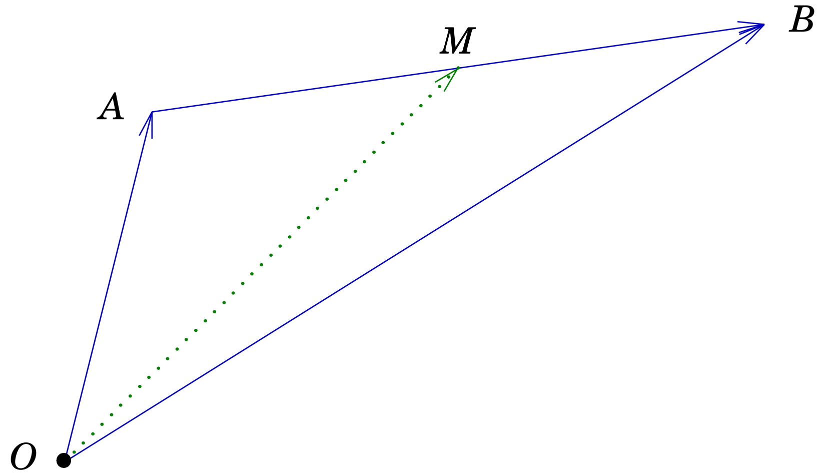
$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \ldots + \boldsymbol{F}_n = \boldsymbol{0}.$$

  For example, a ladder on rough ground against a smooth wall:



Bisectors and other simple geometry

- If M is the mid-point of AB, then $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$.



So

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB},$$

using the triangle law. Now the 'clever' bit. Write $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OA}$; then

$$\overrightarrow{OM} = \tfrac{1}{2}\overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB} = \tfrac{1}{2}\overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{OB},$$

using the triangle law again, or, in the other notation,

$$\boldsymbol{m} = \tfrac{1}{2}\boldsymbol{a} + \tfrac{1}{2}\boldsymbol{b}.$$

- More generally, if M is a fraction f of the way between A and B, then

$$\overrightarrow{OM} = (1-f)\overrightarrow{OA} + f\overrightarrow{OB}, \text{ or } \boldsymbol{m} = (1-f)\boldsymbol{a} + f\boldsymbol{b}.$$

[Special cases: $f = 0, \frac{1}{2}, 1, -1, 2, \ldots$.]