The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL C MODULE, SPRING 2004–2005

COMPUTERISED MATHEMATICAL METHODS IN ENGINEERING

Time allowed TWO hours MOCK

Candidates must NOT start writing their answers until told to do so.

This paper contains FIVE questions which carry equal marks.
Full marks may be obtained for FOUR complete answers.
Credit will be given for the best FOUR answers.

Marks available for sections of questions are shown in brackets in the right-hand margin.

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

You need not record every step in your calculations, but you should give enough detail to demonstrate that you are using an appropriate technique and understand how it works.

DO NOT turn examination paper over until instructed to do so

MOCK EXAM

ADDITIONAL MATERIAL: *Formula sheet*

1 (*a*) Explain why the standard formula should not be used to find the smaller root of the quadratic equation

$$x^2 - Ax + 1 = 0$$

when A is large. Find both roots to calculator accuracy in the case $A = 483$. [8]

(*b*) Find correct to FIVE places of decimals all real roots of the equations

(*i*) $x^4 = x + 1$; [10]

(*ii*) $x = \sin x - 1$. [7]

[You may find it helpful to draw graphs to determine the number and approximate locations of the roots.]

2 (*a*) Find

$$\int_0^1 \frac{1}{x^4 + 1}\, dx$$

to FIVE places of decimals by the use of Simpson's Rule. [16]

(*b*) Explain briefly why

$$\int_0^1 \sqrt{\sin x}\, dx$$

should not be integrated directly by the use of Simpson's Rule. Re-write this integral into a form suitable for such use. [You are NOT asked to evaluate the integral.] [9]

3 (*a*) Use Gaussian elimination to solve the linear equations

$$x - y + 2z - w = -8,$$

$$2x - 2y + 3z - 3w = 20,$$

$$x + y + z = -2,$$

$$x - y + 4z + 3w = 4.$$

[12]

From your solution, write down the determinant of the matrix of coefficients. [3]

(*b*) Briefly explain the power method for finding the largest eigenvalue of a matrix. What are the conditions in which the method can be expected to work well? [6]
Suppose a matrix is believed to have an eigenvalue λ which is close to some real number a. Explain how you could modify the power method to find this λ. [4]

4 (*a*) Describe the Modified Euler Method for solving first-order ordinary differential equations given an initial condition. [5]

(*b*) Explain what is meant by *partial instability* when a given method is applied to solving an ordinary differential equation. Illustrate your account qualitatively by considering the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda y$$

for various values of λ and some given step-length h when a single-step method, such as Modified Euler, is used. [You are not expected to provide numerical values of either the exact solution or the solution as found.] [10]

(*c*) Consider the linear second-order ordinary differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + p(x)\frac{\mathrm{d}y}{\mathrm{d}x} + q(x) = 0.$$

Explain how the equation can be modified to be suitable for solution by a first-order method, such as Modified Euler. Explain further how you could solve the boundary value problem

$$y = 1 \text{ when } x = 0; \;\; y = 0 \text{ when } x = 1$$

for this equation. [10]

5 Consider the heat equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

for $0 \leqslant x \leqslant 1$ and $t > 0$, and with boundary conditions

$$u(0,t) = u(1,t) = 0$$

and initial condition

$$u(x,0) = x\times(1-x).$$

(*a*) Describe a suitable system of finite difference equations for the numerical solution of this parabolic partial differential equation in terms of grid points (x_i, t_j) where $x_i = ih$ and $t_j = jk$ for constants h and k. [10]

(*b*) Write down this system in the case $h = k = \frac{1}{3}$ as far as the terms for $t = 1$. [5]

(*c*) Solve your equations for the given boundary and initial conditions. [10]