HG3MCE—Computerised Mathematical Techniques in Engineering

Problem Class 1

Some exercises on cancellation errors for the problem class of 9th February.

1. Consider the quadratic equation

$$x^2 - 1234x + 1 = 0.$$

Use the standard formula to find the two roots. How many significant figures [depending on your calculator!] do you expect each root to be correct to? Find the smaller root to full calculator accuracy.

[HINT: *Either* note that the product of the roots is 1, *or* do some algebra on the formula, *or* note that we can re-arrange the formula into $x = (1+x^2)/1234$.]

2. For $h = 0.1, 0.01, 0.001, \ldots, 10^{-n}, \ldots$, as far as you think sensible, calculate an approximate derivative of the sine function using the result

$$\cos 1 \approx (\sin(1+h) - \sin 1)/h. \quad \ldots *$$

Draw up a table showing the number of significant figures of agreement between the true value of $\cos 1$ and the approximation, against the exponent n.

Use the Taylor series, $\sin(1+h) = \sin 1 + h \sin' 1 + \frac{1}{2}h^2 \sin'' 1 + \ldots$, to estimate the truncation error in the approximation, and hence estimate [depending on your calculator!] a bound on the total error, as a function of h. [HINT: this will be something like 'truncation error + $10^{-12}/h$'.] For what h is this bound minimised? Compare with your table of results.

What happens if you use the approximation

$$\cos 1 \approx (\sin(1+h) - \sin(1-h))/2h? \quad \ldots *$$

[In case it isn't obvious, *of course* we know how to differentiate $\sin x$, but you may find yourself having to differentiate a function you don't understand, and this exercise may give you some pointers to how to do it numerically and what accuracy you can expect to achieve. To reduce the truncation error, we need more and more complicated formulas involving $\sin(1 \pm 2h)$, $\sin(1 \pm 3h)$ and so on; details beyond the scope of this module.]

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* Note: the $\approx$ in these equations is an 'approximates' symbol, with wavy lines, not an 'equals' symbol. You may need a magnifying glass to spot the difference!