HG3MCE—Computerised Mathematical Techniques in Engineering

Problem Class 2—Solutions

1. (a) $x^4 = x+10$.
Lots of ways of doing this, of course. Clearly [sketch $y = x^4$ and $y = x+10$] there are just two real roots, one between 1 and 2, the other between −1 and −2. One of the simplest is the iteration suggested by $x = (x+10)^{1/4}$. Starting from 0 we find:

$$0 \to 1.7782794100 \to 1.8525522387 \to 1.8554658671 \to 1.8555798855 \to 1.8555843470 \to 1.8555845215 \to 1.8555845284,$$

so the positive real root is $x = 1.855585$, to 6dp. [You don't need, in real life, to show intermediate results, of course. But it's a Good Idea in exams or coursework!]

To find the negative real root, we can do somewhat the same, except that the second square root needs to be negated, or we can use Newton−Raphson:

$$x \leftarrow x - (x^4 - x - 10)/(4x^3 - 1) = (3x^4 + 10)/(4x^3 - 1).$$

Then, for example,

$$-1 \to -2.6 \to -2.062896892 \to -1.781225820 \to -1.702953605 \to -1.697497019 \to -1.697471881 \to -1.697471881,$$

and the negative real root is −1.697472, to 6dp. *Exercise:* Find the complex roots too!

(b) $\sin x = x + \frac{1}{2}$.
If we re-write this as $x = \sin x - \frac{1}{2}$, this is a very simple iteration on a calculator [press the SIN key, subtract 0.5, repeat], which converges from any real starting value [why?]. For example,

$$0 \to -0.5 \to -0.97942554 \to -1.3301772 \to -1.4711906 \to -1.4950435 \to -1.4971321 \to -1.497288 \to -1.4972995 \to -1.4973003 \to -1.4973004 \to -1.4973004,$$

so the root is $x = -1.497300$, to 6dp. Again, there are many other possible ways.

2. We can easily build up a table:

a	$b = \tan a$	$c = \tan b$	$(b^2 - ac)/(2b - a - c)$
4.5	4.6373320545	13.2981924839	4.4977872903
4.4977872903	4.5880410575	8.0004596869	4.4953353523
4.4953353523	4.5345768703	5.5645179022	4.4937809993
4.4937809993	4.5012958582	4.6666714742	4.4934232593
4.4934232593	4.4937019389	4.4996155007	4.4934094769
4.4934094769	4.4934098612	4.4934180059	4.4934094579

The root is $x = 4.4934$ to 4dp. Note that there is no way of avoiding cancellation error in Aitken's process; you will find it very hard to get much more than 6dp from a 12sf calculator. Again, I have given all numbers to full calculator accuracy, and there is no need for this in real life. Indeed, on a programmable calculator, you could easily set it to do the next iteration at the press of a single button, and just sit there pressing that button until it ‘works’.

3. If we look at the difference table,

Year	Precession			
1700	494.50			
		–3473		
1750	459.77		36	
		–3437		–107
1800	425.40		–71	
		–3508		105
1850	390.32		34	
		–3474		–32
1900	355.58		2	
		–3472		
1950	320.86			

You should notice that the second/third differences clearly show the ‘1,–2,1’/‘1,–3,3,–1’ pattern of an error in the table. The error is ‘obviously’ 35 units [juggling 36, 71/2, 34, 107/3, 105/3 and 32 to get a best fit], so the 1800 value ‘ought’ to have been 425.05. [Actually, 425.04 is more likely, and fits almost as well, the error perhaps being to omit the 0 rather than to swap the 0 and the 4.]

With this correction, second differences are very small, and third differences certainly negligible:

Year	Precession			
1700	494.50			
		–3473		
1750	459.77		1	
		–3472		–2
1800	425.05		–1	
		–3473		0
1850	390.32		–1	
		–3474		3
1900	355.58		2	
		–3472		
1950	320.86			

so the 2000 precession is $320.86 - 34.72 = 286.14$ [or 286.16 including 2nd differences at face value].