HG3MCE—Computerised Mathematical Techniques in Engineering

Problem Class 3—Solutions

1. *(a)* Straight exercise in Simpson's Rule. Let $f(x) = \exp(-e^{-x})$. Then we can build up a table [all numbers to calculator accuracy]:

ends	$f(0)+f(1)$	1.060080069
odds	$f(0.5)$	0.5452392119
I_2	$(ends+4\,odds)/6$	0.5401728195
evens	old odds	0.5452392119
odds	$f(0.25)+f(0.75)$	1.082480986
I_4	$(ends+4\,odds+2\,evens)/12$	0.5400402029
evens	old odds + old evens	1.627720198
odds	$f(0.125)+f(0.375)+f(0.625)+f(0.875)$	2.161314135
I_8	$(ends+4\,odds+2\,evens)/24$	0.5400323753

I_8 is clearly correct to 5dp; indeed, we might well have risked simply extrapolating between I_4 and I_2 [giving 0.5400313619]. Applying Richardson extrapolation anyway gives $I \approx 0.5400318535$ or 0.5400 to 4dp. Actually $I \approx 0.5400318623$.

(b) If you simply charge through Simpson's Rule for this, you get into a mess because $\log x \cos x \to -\infty$ as $x \to 0$. Two suggestions: (i) Write $\log x \cos x = \log x\,(\cos x - 1) + \log x$. Now the first term is relatively well-behaved near 0 and the second term can be integrated directly. (ii) Integrate by parts: $\int \log x \cos x \,\mathrm{d}x = \log x \sin x - \int x^{-1} \sin x \,\mathrm{d}x$. Note that the 'bad behaviour' is now concentrated in the integrated term, though at least $\log x \sin x \to 0$ as $x \to 0$. On the other hand, $x^{-1}\sin x$ is, possibly despite appearances, well-behaved for all x—look at its power-series expansion. Everything is now easy, details left to you. Don't forget that $\log x$ is to base e and that trig functions should use radians! You should get −0.9192 to 5dp.

2. For example:

2	1	2	3	1	0	0	0	*(1)*
0	1	–1	–1	0	1	0	0	*(2)*
1	0	3	4	0	0	1	0	*(3)*
2	2	1	3	0	0	0	1	*(4)*
0	1	–4	–5	1	0	–2	0	*(5) = (1)−2×(3)*
0	2	–5	–5	0	0	–2	1	*(6) = (4)−2×(3)*
0	0	3	4	–1	1	2	0	*(7) = (2)−(5)*
0	0	3	5	–2	0	2	1	*(8) = (6)−2×(5)*
0	0	0	1	–1	–1	0	1	*(9) = (8)−(7)*
0	0	3	0	3	5	2	–4	*(10) = (7)−4×(9)*
0	0	1	0	1	5/3	2/3	–4/3	*(11) = (10)/3*
0	1	0	0	0	5/3	2/3	–1/3	*(12) = (5)+4×(11)+5×(9)*
1	0	0	0	–1	1	2	0	*(13) = (3)−3×(11)−4×(9)*
								[Or = (3)−(7)]

We can read off the inverse from equations 13, 12, 11 and 9 [in that order].