HG3MCE—Computerised Mathematical Techniques in Engineering

Problem Class 4—Solutions

1. Clearly we have

$$x = 1+(y+2z+3w)/100,$$
$$y = 2+(z+2w-x)/100,$$
$$z = 3+(w-2x-y)/100,$$
$$w = 4-(3x+2y+z)/100,$$

so, from $x = 1$, $y = 2$, $z = 3$ and $w = 4$, we get successively [to 4dp] $x = 1+(2+6+12)/100 = 1.2$, $y = 2+(3+8-1.2)/100 = 2.098$, $z = 3+(4-2.4-2.098)/100 \approx 2.9950$, $w \approx 3.8921$, $x \approx 1.1976$, $y \approx 2.0958$, $z \approx 2.9940$, $w \approx 3.8922$, $x \approx 1.1976$, and now y, z and w are unchanged to 4dp, so we can take the solution to 3dp to be $x = 1.198$, $y = 2.096$, $z = 2.994$ and $w = 3.892$.

2. The table shows my results. The columns show x, the corresponding calculated value of y, the value of $k_1 = h\sqrt{x-y}$, with $h = 0.2$, the estimated next value $y^* = y+k_1$, the estimated increment $k_2 = h\sqrt{x+h-y^*}$, and the modified next y, $y+\frac{1}{2}(k_1+k_2)$. This last is not really needed, it's the value written down for y in the next row. Of course, with a programmable calculator or a computer program, scarcely any of this needs writing down; if you do write it down [I used a program!], then of course five or so decimal places will do.

x	y	k_1	y^*	k_2	next y
0.0	0.0	0.0	0.0	0.0894427190	0.0447213595
0.2	0.0447213595	0.0788108217	0.1235321812	0.1051604143	0.1367069775
0.4	0.1367069775	0.1026241730	0.2393311505	0.1201114231	0.2480747755
0.6	0.2480747755	0.1186465717	0.3667213472	0.1316478108	0.3732219667
0.8	0.3732219667	0.1306565013	0.5038784680	0.1408717902	0.5089861124
1.0	0.5089861124				

The resulting estimate, $y(1) \approx 0.509$, is not that good. If you try again with $h = 0.01$, you find $y(1) \approx 0.5123$, but that's too much work for pencil-and-paper!